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On invertibility of some operator sums

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Abstract

We study invertibility of some sums of linear bounded operators on Hilbert space (Theorem 1). A criterion on invertibility of sums of projections is found. Some equivalent conditions on invertibility of difference of two projections are obtained. We prove that block projection operators preserve invertibility of positive operators. We present three corollaries from Theorem 1; it is shown for instance that if A, B ∈ B(H) are nonnegative and A - B is invertible, then A + B is also invertible. We also prove the following result: Let X, Y ∈ B(H) be self-adjoint operators, X ≥ 0 and X ≤ Y ≤ X. If Y is invertible, then X is also invertible. It is shown that for unitary operators U, V the operator U + V is invertible if and only if {double pipe}U - V{double pipe} < 2. © 2012 Pleiades Publishing, Ltd.

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Keywords

Hilbert space, invertibility, isometry, linear bounded operator, projection, unilateral shift, unitary operator